

On a Theory for Nonminimal Gravitational-Electromagnetic Coupling Consistent with Observational Data

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The idea that the seed primordial magnetic fields can be explained by the nonminimal coupling between gravitational and electromagnetic fields is discussed. The predicted values of the magnetic field of the spiral galaxies are in agreement with the observations.

1 Introduction

The principle of minimal coupling (PMC) ¹, incorporated into Einstein's theory, states that the matter fields do not couple directly to curvature. In spite of its simplicity, this principle may not lead to the correct theory of the interaction between gravitation and matter fields. Furthermore, if derivatives, higher than the first, occur in a special relativistic Lagrangian, the PMC does not lead to a unique Lagrangian in the presence of the gravitational field. Consequently, we examine a more general approach, such as that offered by the nonminimal coupling (NMC) between gravitation and matter fields, which does not necessarily violate the equivalence principle ⁴.

Nonminimal gravitational-electromagnetic coupling, as embodied in the S-B conjecture ^{2,3}, indicates the empirical relation between the angular momentum \mathbf{L} and the magnetic dipole moment \mathbf{m}

$$\mathbf{m} \cong \beta \left[\frac{\sqrt{G}}{2c} \right] \mathbf{L}, \quad (1)$$

where G is the Newtonian constant of gravitation, c is the speed of light and β is a constant.

Although, we do not yet have a satisfactory gravitational theory which encompasses the S-B conjecture, there is an increasing amount of evidence in its support. When we consider the magnetic field of astronomical objects, we note that the values calculated from Eq.(1) are in agreement with the observed values. Sirag ⁵ compared the predictions of Eq.(1) with the observed values of the ratio of the magnetic moment to the angular momentum for the Earth, Sun, the star 78 Vir, the Moon, Mercury, Venus, Jupiter, Saturn, and the neutron star Her X-1. The minimum values for β for these objects were: 0.12, 0.02, 0.02, 0.11, 0.37, 0.04, 0.03, 0.03, and 0.07, respectively. Excluding the star 78 Vir, the maximum value for β

was 0.77 for the planet Mercury. Woodward⁶ examined the S-B conjecture in the context of pulsar gyromagnetic ratios, for short-period pulsars. He found that: 1) β is not the same for all pulsars; 2) young pulsars evolve with their individual value of β , constant for a discernible period of time; and 3) β lies in the range 0.001 to 0.01.

2 Origin of the Primordial Magnetic Fields in the Universe due to NMC

The origin of magnetic fields in the Universe is one of the major problems in astrophysics. Magnetic fields of $\sim 10^{-6}$ G are observed in galaxies such as our own. Many theories have been proposed over the last half century for the source of the magnetic field. Essentially all of them have been shown to be unsatisfactory⁷.

We suggest⁸ that the observed 10^{-6} G magnetic field in the spiral galaxies could originate from the angular momentum of the protogalaxies through nonminimal gravitational-electromagnetic coupling as manifested by the S-B conjecture.

Basically, we consider that the angular momentum of galaxies was acquired during the protogalaxy stage through the tidal torques by neighboring protogalaxies⁹. We studied a protogalaxy with total mass $M \sim 10^{13} M_{\odot}$, corresponding to a spiral galaxy possessing a halo of dark matter ~ 10 times the mass of the luminous matter $M_L \sim 10^{12} M_{\odot}$.

The angular momentum of the protogalaxy increased until the protogalaxies became sufficiently far apart so that the protogalaxy decoupled from the other protogalaxies, preserving its angular momentum \mathbf{L} , acquired from the tidal interaction with the other protogalaxies. Up to the time of decoupling, the mean density of the protogalaxy was roughly that of the Universe, $\rho(z) = (1+z)^3 \rho_0$, where ρ_0 is the present matter density of the Universe ($\rho_0 \sim 1.057 \times 10^{-29}$ g cm⁻³ with $H_0 = 75$ km s⁻¹ Mpc⁻¹).

The radius of the protogalaxy $R(z)$ is then $R(z) = [(3/4\pi) M \rho(z)^{-1}]^{1/3}$ and, accordingly, the angular momentum $L(z)$ is

$$L(z) \cong \frac{2}{5} \lambda_{\text{med}} [GM^3 R(z)]^{1/2}, \quad (2)$$

where λ_{med} is a parameter obtained from numerical simulations¹⁰. The magnetic field in the vicinity of the protogalaxy is obtained from the relation $\mathbf{B}_{\text{NMC}}(z) \cong \mathbf{m}(z)R(z)^{-3}$. Since the magnetic field of the magnetic dipole is frozen into the plasma of the galaxy, which collapses to a present radius $R_L \simeq 10$ kpc $\simeq 3.1 \times 10^{22}$ cm, the present magnetic field is then obtained at the radius R_L from $\mathbf{B}_0(R_L, z_d) \cong \mathbf{B}_{\text{NMC}}(z_d)[R(z_d)/R_L]^3$. There is an additional amplification due to the differential rotation which ranges from 10 to 100.

Hence, for a decoupling redshift $z_d \leq 10$, we obtain the present magnetic field

$$B_0(R_L, z_d) \sim 10^{-6} - 10^{-5} \text{ G}, \quad (3)$$

where we use $\beta = 0.1$ in Eq.(1).

For $\beta = 0.01$ with an amplification due to differential rotation of 100, we obtain the same values of $B_0(R_L, z_d)$ as given in relation (3). In both cases, the results are consistent with the ranges obtained by Sirag and Woodward.

One might imagine that the process described here could explain the dark matter problem in galaxies as well. The idea is that matter spinning around the galaxy at a distance of approximately 50 kpc would be subject to an extra attraction force due to the magnetic field generated by its own rotation and to the rotation of the galaxy; both via Eq.(1). However, we found that the intensity of the extra force is of order 10^{-40} dyne. This intensity is too small to account for the rotation curves without considering dark matter.

3 Conclusions

The increasing amount of evidence in favor of the S-B conjecture in the astrophysical domain indicates that perhaps gravitation is not minimally coupled to electromagnetic fields. In general, the NMC is believed to be possible only in regions of strong gravitational fields. The evidence, however, points in the direction that NMC may exist even though strong gravitational fields are not involved.

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References

1. P. G. Bergmann, Intern. J. Theor. Phys. **1**, 25 (1968); H. F. M. Goenner, Found. Phys. **14**, 865 (1984); M. Novello and L. A. R. Oliveira, Rev. Bras. Fis. **17**, 432 (1987).
2. A. Schuster, Proc. R. Inst. **13**, 273 (1890); Proc. Phys. Soc. London **24**, 121 (1912).
3. P. M. S. Blackett, Nature **159**, 658 (1947); P. M. S. Blackett, Philos Trans. R. Soc. London A **245**, 309 (1952).
4. A. J. Accioly and U. F. Wichoski, Class. Quantum Grav. **7**, L139 (1990).
5. S.-P. Sirag, Nature **278**, 535 (1979).
6. J. F. Woodward, Found. Phys. **19**, 1345 (1989).
7. G. Field, in Proceedings of the International Congress of Plasma Physics, Foz de Iguaçu, 1994 (AIP, New York, 1995); R. Opher, *ibid*; J. H. Piddington, Aust. J. Phys. **23**, 731 (1970); R. M. Kulsrud, in IAU Symp. 140, Galactic and Intergalactic Magnetic Fields, ed. R. Beck *et al.* (Kluwer, Dordrecht, 1990), p. 527; J. H. Piddington, *Cosmic Electrodynamics*, (Krieger, Malabar, 1981), 2nd ed..
8. R. Opher and U. F. Wichoski, Phys. Rev. Lett. **78**, 787 (1997).
9. P. J. E. Peebles, Astrophys. J. **155**, 393 (1969); S. D. M. White, Astrophys. J. **286**, 38 (1984); G. Efstathiou and B. J. T. Jones, Mon. Not. R. Astron. Soc. **186**, 133 (1979); J. Barnes and G. Efstathiou, Astrophys. J. **319**, 575 (1987).

10. T. Padmanabhan, *Structure Formation in the Universe*, (Cambridge University Press, Cambridge, 1993).